SM 3H

Unit 2 Test Review

Key

2017-2018

Name _____ Date _____ Period _____

Write the expression in reduced form. Show work.

1. $\frac{45x^5}{9x^2}$ $= 5x^3$

2. $\frac{8y^2-18y}{28y-63}$ $2y(4y-9)$ $7(4y-9)$ $\frac{2y}{7}$

3. $\frac{12x^2-6x+3}{8x^3+1}$ $3(4x^2-2x+1)$ $(2x+1)(4x^2-2x+1)$ $\frac{3}{2x+1}$

4. $\frac{5x^2-7x-6}{5x^2+18x+9}$ $(5x+3)(x-2)$ $(5x+3)(x+3)$ $\frac{x-2}{x+3}$

Simplify each expression by multiplying or dividing. Show work.

5. $\frac{10}{x+3} \cdot \frac{x^2-9}{40x-30}$ $(x+3)(x-3)$ $10(4x-3)$ $\frac{x-3}{4x-3}$

6. $\frac{x^2+x-42}{x^2+5x-14} \cdot \frac{13x^3+26x^2}{x^2-4x-12}$ $(x+7)(x-6)$ $13x^2(x+2)$ $(x+7)(x-2)$ $(x-6)(x+2)$ $\frac{13x^2}{x-2}$

7. $\frac{6x^3}{5x^2-5} \div \frac{10xy^2}{(x-1)^2}$ $\frac{3\cdot 2}{}$ $\frac{(x-1)^2}{10xy^2}$ $\frac{5}{}$ $5(x+1)(x-1)$ $\frac{3x^2(x-1)}{25y(x+1)}$

8. $\frac{64x^2-1}{5y^2-40y} \div \frac{8x^2-23x-3}{y^2-64}$ $\frac{(8x+1)(8x-1)}{5y(y-8)} \cdot \frac{(y+8)(y-8)}{(8x+1)(x-3)}$ $\frac{(8x-1)(y+8)}{5y(x-3)}$

9. $\frac{x+y}{6} \cdot \frac{3}{x^2-y^2}$ $\frac{x+y}{2\cdot 3} \cdot \frac{3}{(x+y)(x-y)}$ $\frac{1}{2(x-y)}$

10. $\frac{y^4-81}{xy+4y+3x+12} \div y^2+9$ $\frac{(y^2-9)(y^2+9)}{(x+4)(x+3)} \cdot \frac{1}{y^2+9}$ $\frac{(y+3)(y-3)}{(x+4)(y+3)} = \frac{y-3}{x+4}$

Perform the indicated operation, if possible simplify. Show work.

$\frac{(c+3)}{(c+3)}\frac{10}{c-4}+\frac{8}{c+3}\frac{(c-4)}{(c-4)}$

$\frac{10(c+3)+8(c-4)}{(c-4)(c+3)}$

$\frac{10c+30+8c-32}{(c-4)(c+3)}$

$\frac{18c-2}{(c-4)(c+3)}$

12. $\frac{2x-3}{x+6}+\frac{8}{1}\cdot\frac{x+6}{x+6}$

$\frac{2x-3+8(x+6)}{x+6}$

$\frac{2x-3+8x+48}{x+6}$

$\frac{10x+45}{x+6}$

13. $\frac{(2p-3)}{p^2-5p+6}\frac{(p+3)}{(p+3)}-\frac{5}{p^2-9}\frac{(p-2)}{(p-2)}$

$(p+3)(p-3)(p-2)\quad(p-3)(p+3)(p-2)$

$\frac{(2p-3)(p+3)-5(p-2)}{(p+3)(p-3)(p-2)}$

$\frac{2p^2+3p-9-5p+10}{(p+3)(p-3)(p-2)}$

$\frac{2p^2-2p+1}{(p+3)(p-3)(p-2)}$

14. $\frac{x^2-2x+8}{x^2-x-12}-\frac{3x-2}{x+3}$

$\frac{(x-4)(x+3)}{}$

$\frac{x+2-3x-2}{x+3}$

$\frac{-2x}{x+3}$

$\frac{(x+1)}{15.}\frac{5x}{(x+1)4x-3}-\frac{8}{x+1}\cdot\frac{4x-3}{4x-3}$

$\frac{5x(x+1)-8(4x-3)}{(x+1)(4x-3)}$

$\frac{5x^2+5x-32x+24}{(x+1)(4x-3)}$

$\frac{5x^2-27x+24}{(x+1)(4x-3)}$

16. $\frac{2-5m}{m-9}+\frac{4m-5}{9-m}\cdot\frac{(-1)}{(-1)}$

$\frac{2-5m-4m+5}{m-9}$

$\frac{-9m+7}{m-9}$

Solve the equation algebraically. Identify any extraneous solutions. Show work.

$x\neq5$

17. $\left(\frac{5}{x-5}=\frac{x}{x-5}-1\right)\frac{x-5}{1}$

$5=x-x+5$

$5=5$

All real numbers except $x\neq5$.

$x\neq-2$

18. $\left(\frac{x}{x+2}+x=\frac{5x+8}{x+2}\right]\frac{x+2}{1}$

$x+x(x+2)=5x+8$

$x+x^2+2x=5x+8$

$x^2+3x=5x+8$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$x=4,\ x=-2$

↓ extraneous

$d\neq7,2$

19. $\left[\frac{1}{d-7}+\frac{d}{d-2}=\frac{5}{d^2-9d+14}\right]\frac{(d-7)(d-2)}{1}$

$(d-7)(d-2)$

$d-2+d(d-7)=5$

$d-2+d^2-7d=5$

$d^2-6d-7=0$

$(d-7)(d+1)=0$

$d=7,\ d=-1$

↓ extraneous

$x \geq -2/5$

20. $\sqrt{5x+2}+8=11$

$(\sqrt{5x+2})^2=(3)^2$

$5x+2=9$

$5x=7$

$x=7/5$

$x \geq 0$

21. $-2\sqrt{\frac{x}{5}}+4=14$

$\frac{-2\sqrt{\frac{x}{5}}}{-2}=\frac{10}{-2}$

$\sqrt{\frac{x}{5}}=-5$

no solution, $\sqrt{\frac{x}{5}}$ can not be a negative #

22. $5\sqrt[3]{x}-12=8$ no restrictions

$5\sqrt[3]{x}=20$

$(\sqrt[3]{x})^3=(4)^3$

$x=64$

no restrictions

23. $\sqrt[3]{x-6}+2=-1$

$(\sqrt[3]{x-6})^3=(-3)^3$

$x-6=-27$

$x=-21$

no restrictions

24. $(x+5)^{\frac{2}{3}}-8=41$

$[(x+5)^{2/3}]^{3/2}=[49]^{3/2}$

$x+5=\pm(49)^{3/2}$

$x+5=\pm 343$

$x=338, \ x=-348$

$x \geq 1$

25. $4(x-1)^{\frac{3}{4}}-7=101$

$4(x-1)^{3/4}=108$

$[(x-1)^{3/4}]^{4/3}=(27)^{4/3}$

$x-1=81$

$x=82$

26. Write out the first five terms of the sequence. Show work!

$a_n=n^3-n^2$

$a_1=(1)^3-(1)^2=0$

$a_2=(2)^3-(2)^2=8-4=4$

$a_3=(3)^3-(3)^2=27-9=18$

$a_4=(4)^3-(4)^2=64-16=48$

$a_5=(5)^3-(5)^2=125-25=100$

0, 4, 18, 48, 100

27. Find the first six terms of the sequence. Show work!

$a_1=-5, \quad a_n=4\cdot a_{n-1}$

$a_1=-5$

$a_2=4\cdot(-5)=-20$

$a_3=4(-20)=-80$

$a_4=4(-80)=-320$

$a_5=4(-320)=-1280$

$a_6=4(-1280)=-5120$

$-5, -20, -80, -320, -1280, -5120$

28. Find the explicit rule for the nth term of the arithmetic sequence.

$-18, -9, 0, 9, \ldots$

$a_n=-18+(n-1)(9)$

$=-18+9n-9$

$a_n=9n-27$

29. Find the explicit rule for the nth term of the geometric sequence.

8, 32, 128, 512, ...

$a_n = 8(4)^{n-1}$

30. Find the recursive rule for the nth term of the arithmetic sequence.

4, 5, 6, 7, ...

$a_1 = 4,\ a_n = a_{n-1} + 1,\ n \geq 2$

31. A certain species of tree grows an average of 4.7 cm per week. Write an explicit rule for the sequence that represents the weekly height of this tree in centimeters if the measurements begin when the tree is 7 meters tall. (1 meter = 100 centimeters)

$a_1 = 700 + 4.7n$

or $a_1 = 4.7n + 700$

32. Write the series using summation notation.

$4 - 16 + 64 - 256 + \cdots$

$\sum_{k=1}^{\infty} 4(-4)^{k-1}$

33. Find the common difference of the given arithmetic sequence.

$-12, -7, -2, 3, 8$

$d = 5$

34. Find the sum of the arithmetic sequence using the formula. Show work!

22, 26, 30, 34, ..., 46

$S = \frac{n}{2}(a_1 + a_n)$

$S = \frac{7}{2}(22 + 46) = \frac{7}{2}(68) = 238$

$46 = 22 + (n-1)(4)$

$24 = 4(n-1)$

$6 = n - 1$

$7 = n$

35. Find the common ratio of the given geometric sequence.

$45, -15, 5, -\frac{5}{3}, \frac{5}{9}$

$r = \frac{-15}{45} = \frac{-1}{3}$

36. Find the sum of the geometric sequence using the formula. Show work!

$2, -8, 32, -128, 512$

$S = \frac{a(1-r^n)}{1-r}$

$S = \frac{2(1-(-4)^5)}{1-(-4)} = \frac{2050}{5} = 410$

37. Find the sum of the first n terms of the sequence. Show work!

$19, 26, 33, 40, \ldots; n = 11$

$d = 7$

$\frac{11}{2}(2(19) + (10)(7)) = \frac{11}{2}(38 + 70) = \frac{11}{2}(108) = 594$

38. Determine whether the infinite geometric series converges. Explain why or why not. (Show work!) If the series converges, determine the limit.

$11 + 55 + 275 + 1375 + \cdots$ $r=5$, diverges $r>1$

39. Determine whether the infinite geometric series converges. Explain why or why not. (Show work!) If the series converges, determine the limit.

$48 + 16 + \frac{16}{3} + \frac{16}{9} + \cdots$ $r = \frac{1}{3}$, $|r|<1$ converges, so, limit $= \frac{a}{1-r}$ $\frac{48}{1-1/3} = \frac{48}{2/3} = 72$

40. An auditorium has 30 rows with 10 seats in the first row, 12 in the second row, 14 in the third row, and so forth. How many seats are in the auditorium? Use the formula. Show work!

$n=30$ $a_1 = 10$, $d = 2$

$S = \frac{30}{2}(2(10) + 29(2)) = 15(20+58) = 15(78) = 1170$

41. Expand the binomial (use your method of choice). Show work!

$(x-y)^5$ 1, 5, 10, 10, 5, 1

$x^5 + 5x^4(-y) + 10x^3(-y)^2 + 10x^2(-y)^3 + 5x(-y)^4 + (-y)^5$

$x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

42. Evaluate using the formula. Show work!

$\binom{11}{9}$ $\frac{11!}{9!\,2!} = \frac{11 \cdot 10 \cdot 9!}{9! \cdot 2!} = 55$

43. Evaluate using the formula. Show work!

$\binom{12}{12}$ $\quad \frac{12!}{12!\,0!} = \boxed{1}$

44. Find the coefficient of the given term in the binomial expansion. Show work!

x^3y^{11} term, $(x+y)^{14}$

$\binom{14}{11} = 364$ $\quad$ or $\binom{14}{3} = 364$

45. Expand the binomial using the Binomial Theorem. Show work!

$(5x-4)^4$ $\quad$ 1, 4, 6, 4, 1

$a = 5x$

$b = -4$

$a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(5x)^4 + 4(5x)^3(-4) + 6(5x)^2(-4)^2 + 4(5x)(-4)^3 + (-4)^4$

$\boxed{625x^4 - 2000x^3 + 2400x^2 - 1280x + 256}$